A BETTER WAY TO SOLVE THE INVERSE INVOLUTE FUNCTION

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The involute curve is often used to describe gear tooth shapes. Although the curve is easily formed and the involute easily found, solving the inverse involute is more difficult. The involute function is

$$\varepsilon = \tan\phi - \phi$$
$$= \text{inv}\phi$$

The inverse involute function $\phi = \text{inv}^{-1}\varepsilon$ is frequently encountered in gear designs. If pressure angle ϕ is known, finding invϕ is easy. But when $\text{inv}^{-1}\varepsilon$ is needed, an approximate value of ϕ is usually interpolated from an involute table. Often, this is not sufficiently accurate.

An iterative method for solving the inverse-involute function was presented in "Solving the Involute Function More Accurately" (Aug. 25, 1988, p. 119). The method presented here does not require iterations or initial guesses. Hence, it can be written on one line into programs that require the involute for a numerical solution. The equations are:

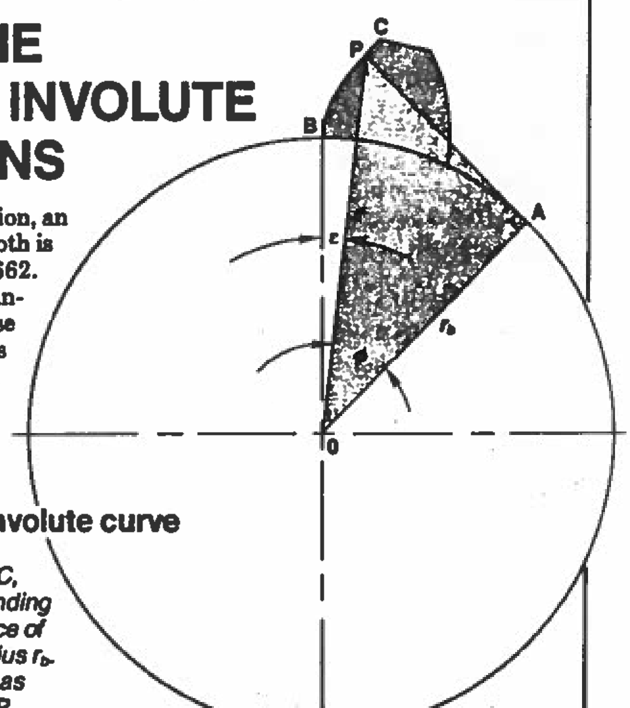
$$\phi \approx (3\varepsilon)^{1/3} - \frac{2}{5}\varepsilon + \frac{9}{175}3^{2/3}\varepsilon^{5/3} - \frac{2}{175}3^{1/3}\varepsilon^{7/3} + \ldots \quad (1)$$

for $|\varepsilon| < 1.8$

$$\phi \approx \frac{5\pi}{12} + (7 - 4\sqrt{3})\zeta - (388 - 224\sqrt{3})\zeta^2$$
$$+ \frac{1}{3}(323{,}565\sqrt{3} - 560{,}431)\zeta^3$$
$$- \frac{1}{3}(97{,}383{,}044\sqrt{3}$$

USING THE INVERSE INVOLUTE EQUATIONS

During a gear inspection, an involute ε of a gear tooth is measured at 0.024662. What is the pressure angle of the gear? Because the value for ε is less than 1.8, Equation 1 is used, and ϕ is calculated as 23.5°.



Generating an involute curve

The involute curve, BC, is generated by unwinding an arc from the surface of a base circle with radius r_b. By doing so, arc AB has the same length as AP.

$$- 168{,}672{,}380)\zeta^4 + \ldots \quad (2)$$

for $1.8 < |\varepsilon| < 5$, and

$$\phi \approx \frac{\pi}{2} - \frac{1}{\varepsilon} + \frac{\pi}{2\varepsilon^2} - \left[\left(\frac{\pi}{2}\right)^2 + \frac{2}{3}\right]\frac{1}{\varepsilon^3} + \left[\left(\frac{\pi}{2}\right)^3 + \pi\right]\frac{1}{\varepsilon^4} - \left[\left(\frac{\pi}{2}\right)^4 + \pi^2 + \frac{13}{15}\right]\frac{1}{\varepsilon^5} + \ldots \quad (3)$$

for $|\varepsilon| > 5$. In the equations, $\zeta = \varepsilon - \mathrm{inv}(5\pi/12)$.

From the equations, the error in ϕ is determined by the last term in each string. Maximum equation error is less than 3′19.72″ or 0.001 rad.

For gear design and analysis, pressure angle ϕ is less than 45°. The error in ϕ using the first equation is less than 2.9″ or 0.00001 rad. By using only the first two terms in the same equation, maximum error in ϕ is less than 26′40.68″ or 0.0078 rad, which is comparable to the error produced with the linear-interpolation method. ■

CALCULATING WIRE DESIGN FACTORS

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Many design engineers frequently refer to a copper wire table to find wire diameter, resistance, and weight. Engineers who work with inductors, power supplies, cables and interconnections may need this data many times a day. Though published copper wire tables are available from most wire manufacturers and other reference sources, it is convenient to have an algorithm or equation handy to calculate these parameters when a reference manual is not available.

An equation already exists with an accuracy sufficient to meet most engineering needs. The equation is simple enough to be entered into a programmable or equation-solving calculator, such as the HP-27S, providing a hand-held or portable wire table.

Wire diameter nearly doubles every six wire sizes; therefore, wire size may be described by an exponential model of the form: $y = Be^{(Mx)}$, where y = wire diameter and x = wire gage, e = natural logarithm, and B and M are constants. A linear regression analysis performed on representative values from a standard wire table yields values of 0.324883 for B, and -0.115947 for M. Substituting in the original equation gives:

$$y = 0.324883e^{-0.115947x}$$

Transforming the equation to solve wire gage gives: $x = [\ln(y/0.324883)]/(-0.115947)$. Next, wire cross-sectional area A_w is calculated from .

$$A_w = (\pi/4)(y^2) = 0.082898e^{-0.231894x}$$

The density of copper, ρ, is approximately 0.3212 lb/in.^{3}, and can be used to calculate the weight of a length of wire of a given gage:

$$W = PlA_w = 0.3212l\,(0.082898e^{-0.231894x})$$

Finally, the resistivity a of copper, about 0.67879 μΩ-in., can be used to calculate the resistance R of a length of wire of a given gage:

$$R = al/A_w = \frac{(0.67879 \times l10^{-6})}{(0.082898e^{-0.231894x})}$$

These equations can be solved for any of the independent variables. Solutions tested all agree with published tables within a fraction of a percent. The values for density and resistivity are for copper at room temperature. Another helpful feature is that values for other materials, or values at other temperatures can be substituted. ■